International Journal of Engineering Sciences &Research Technology

(A Peer Reviewed Online Journal)
Impact Factor: 5.164

IJESRT

Chief Editor
Dr. J.B. Helonde

Executive Editor
Mr. Somil Mayur Shah

IJESRT
INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

OBSERVATION ON THE POSITIVE PELL EQUATION $y^2 = 35x^2 + 46$

T.R.Usha Rani*[1], V.Bahavathi[2] & K.Sridevi[3]
[*1]Assistant professor, Department of Mathematics, Shrimati Indira Gandhi College, Trichy-620 002, Tamil nadu, India
[2]Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Trichy-620 002,Tamil nadu, India
[3]PG Scholar, Department of Mathematics, Shrimati Indira Gandhi College, Trichy-620 002, Tamil nadu, India

DOI: Will get Assigned by IJESRT Team

ABSTRACT

The binary quadratic equation represented by the positive pellian $y^2 = 35x^2 + 46$ is analyzed for its distinct integer solutions. A few interesting relation among the solutions are given. Employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbolas and parabolas

KEYWORDS: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solution.

1. INTRODUCTION

The binary quadratic Diophantine equations are rich is variety. The binary quadratic equation of the form $y^2 = Dx^2 + 1$.where D is non square positive integer has been satisfied by various mathematician for its non-trivial integral solution. When D takes different integral values[1-4].In [5-11] the binary quadratic non-homogeneous equation representing hyperbolas respectively are studied for their non-zero integral solutions. This communication concerns with yet another binary quadratic equation given by $y^2 = 35x^2 + 46$. The recurrence relation satisfied by the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

2. METHOD OF ANALYSIS

The positive Pell equation representing hyperbola under consideration is

$$y^2 = 35x^2 + 46 \tag{1}$$

The smallest positive integer solutions of (1) are

$$x_0 = 1,\ y_0 = 9$$

To obtain the order solution of (1), consider the pellian equation

$$y^2 = 35x^2 + 1 \tag{2}$$

Whose initial solution is given by

$$\tilde{x}_0 = 1,\ \tilde{y}_0 = 6$$

The general solution $(\tilde{x}_n, \tilde{y}_n)$ of (2) is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{35}} g_n,\ \tilde{y}_n = \frac{1}{2} f_n$$

Where

$$f_n = (6+\sqrt{35})^{n+1} + (6-\sqrt{35})^{n+1}$$

$$g_n = (6+\sqrt{35})^{n+1} - (6-\sqrt{35})^{n+1}, n = -1,0,1,\ldots..,$$

IJESRT is licensed under a Creative Commons Attribution 4.0 International License.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the other integer solution to (1) are given by

$$x_{n+1} = \frac{1}{2} f_n + \frac{9}{2\sqrt{35}} g_n$$

$$y_{n+1} = \frac{9}{2} f_n + \frac{35}{2\sqrt{35}} g_n$$

The recurrence relation satisfied by the solution x and y are given by

$$x_{n+3} + x_{n+1} - 12x_{n+2} = 0$$

$$y_{n+3} + y_{n+1} - 12y_{n+2} = 0$$

Some numerical examples of x_n and y_n satisfying (1) are given in the Table: 1 below:

Table 1: Numerical Examples

n	x_n	y_n
0	1	9
1	15	89
2	179	1059
3	2133	150369
4	25417	1654059

From the above table, we observe some interesting relations among the solutions which are presented below:

- Both x_n and y_n values are odd .

1. Relations among the solutions are given below.

- $x_{n+2} - y_{n+1} - 6x_{n+1} = 0$
- $x_{n+3} - 12y_{n+1} - 71x_{n+1} = 0$
- $y_{n+2} - 6y_{n+1} - 35x_{n+1} = 0$
- $y_{n+3} - 71y_{n+1} - 420x_{n+1} = 0$
- $x_{n+3} - 12x_{n+2} + x_{n+1} = 0$
- $y_{n+2} - 6x_{n+2} + x_{n+1} = 0$
- $y_{n+3} - 71x_{n+2} + 6x_{n+1} = 0$
- $y_{n+2} - x_{n+3} + x_{n+1} = 0$
- $12y_{n+3} - 71x_{n+3} + x_{n+1} = 0$
- $6y_{n+3} - 71y_{n+2} - 35x_{n+1} = 0$
- $6x_{n+3} - y_{n+1} - 71x_{n+2} = 0$
- $6y_{n+2} + y_{n+1} - 35x_{n+2} = 0$
- $71y_{n+2} - 6y_{n+1} - 35x_{n+2} = 0$
- $71y_{n+3} - y_{n+1} - 420x_{n+3} = 0$
- $y_{n+3} - 12y_{n+2} + y_{n+1} = 0$
- $y_{n+2} - x_{n+3} + 6x_{n+2} = 0$
- $y_{n+3} - 6x_{n+3} + x_{n+2} = 0$

IJESRT is licensed under a Creative Commons Attribution 4.0 International License.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

- $y_{n+3} - 6y_{n+2} + 35x_{n+2} = 0$
- $6y_{n+3} - y_{n+2} - 35x_{n+3} = 0$
- $y_{n+3} - y_{n+1} - 70x_{n+2} = 0$

2. Each of the following expression represents a Nasty Number:

- $\frac{6}{23}[46 + 9y_{2n+2} - 35x_{2n+2}]$
- $\frac{6}{23}[46 + 9x_{2n+3} - 89x_{2n+2}]$
- $\frac{3}{46}[184 + 3x_{2n+4} - 353x_{2n+2}]$
- $\frac{3}{23}[92 + 3y_{2n+3} - 175x_{2n+2}]$
- $\frac{6}{1633}[3266 + 9y_{2n+4} \_ 6265x_{2n+2}]$
- $\frac{6}{23}[46 + 15y_{2n+2} - y_{2n+3}]$
- $\frac{1}{46}[552 + 179y_{2n+2} - y_{2n+4}]$
- $\frac{6}{23}[46 + 89x_{2n+4} - 1059x_{2n+3}]$
- $\frac{6}{23}[46 + 89y_{2n+3} - 525x_{2n+3}]$
- $\frac{6}{138}[276 + 89y_{2n+4} - 6265x_{2n+3}]$
- $\frac{3}{23}[92 + 353y_{2n+3} - 175x_{2n+4}]$
- $\frac{6}{23}[46 + 1059y_{2n+4} - 6265x_{2n+4}]$
- $\frac{6}{23}[46 + 179y_{2n+3} - 15y_{2n+4}]$
- $\frac{6}{138}[276 + 89y_{2n+2} - 35x_{2n+3}]$
- $\frac{6}{1633}[3266 + 1059y_{2n+2} - 35x_{2n+4}]$

3. Each of the following expression represents a cubical Integer:

- $\frac{1}{23}[9y_{3n+3} - 35x_{3n+3} + 27y_{n+1} - 105x_{n+1}]$
- $\frac{1}{23}[9x_{3n+4} - 89x_{3n+3} + 27x_{n+2} - 267x_{n+1}]$
- $\frac{1}{92}[3x_{3n+5} - 353x_{3n+3} + 9x_{n+3} - 1059x_{n+1}]$
- $\frac{1}{46}[3y_{3n+4} - 175x_{3n+3} + 9y_{n+2} - 525x_{n+1}]$

IJESRT is licensed under a Creative Commons Attribution 4.0 International License.

- $\frac{1}{1633}\left[9y_{3n+5}-6265x_{3n+3}+27y_{n+2}-18795x_{n+1}\right]$
- $\frac{1}{138}\left[89y_{3n+3}-35x_{3n+4}+267y_{n+1}-105x_{n+2}\right]$
- $\frac{1}{1633}\left[1059y_{3n+3}-35x_{3n+5}+3177y_{n+1}-105x_{n+3}\right]$
- $\frac{1}{23}\left[15y_{3n+3}-y_{3n+4}+45y_{n+1}-3y_{n+2}\right]$
- $\frac{1}{276}\left[179y_{3n+3}-y_{3n+5}+573y_{n+1}-3y_{n+3}\right]$
- $\frac{1}{23}\left[89x_{3n+5}-1059x_{3n+4}+267x_{n+3}-3177x_{n+2}\right]$
- $\frac{1}{23}\left[89y_{3n+4}-525x_{3n+4}+267y_{n+2}-1575x_{n+2}\right]$
- $\frac{1}{138}\left[89y_{3n+5}-6265x_{3n+4}+267y_{n+3}-18795x_{n+2}\right]$
- $\frac{1}{46}\left[353y_{3n+4\backslash}-175x_{3n+5}+1059y_{n+2}-525x_{n+3}\right]$
- $\frac{1}{23}\left[1059y_{3n+5}-6265x_{3n+5}+3177y_{n+3}-18795x_{n+3}\right]$
- $\frac{1}{23}\left[179y_{3n+4}-15y_{3n+5}+537y_{n+2}-45y_{n+3}\right]$

4. Each of the following expression represents a bi-quadratic Integer:

- $\frac{1}{23}\left[9y_{4n+4}-35x_{4n+4}+36y_{2n+2}-140x_{2n+2}+138\right]$
- $\frac{1}{23}\left[9x_{4n+5}-89x_{4n+4}+36x_{2n+3}-356x_{2n+2}+138\right]$
- $\frac{1}{92}\left[3x_{4n+6}-353x_{4n+4}+12x_{2n+4}-1412x_{2n+2}+552\right]$
- $\frac{1}{46}\left[3y_{4n+5}-175x_{4n+4}+12y_{2n+3}-700x_{2n+2}+276\right]$
- $\frac{1}{1633}\left[9y_{4n+6}-6265x_{4n+4}+36y_{2n+4}-25060x_{2n+2}+9798\right]$
- $\frac{1}{138}\left[89y_{4n+4}-35x_{4n+5}+356y_{2n+2}-140x_{2n+3}+828\right]$
- $\frac{1}{1633}\left[1059y_{4n+4}-35x_{4n+6}+4236y_{2n+2}-140x_{2n+4}+9798\right]$
- $\frac{1}{23}\left[15y_{4n+4}-y_{4n+5}+60y_{2n+2}-4y_{2n+3}+138\right]$
- $\frac{1}{276}\left[179y_{4n+4}-y_{4n+6}+716y_{2n+2}-4y_{2n+4}+1656\right]$
- $\frac{1}{23}\left[89x_{4n+6}-1059x_{4n+5}+356x_{2n+4}-4236x_{2n+3}+138\right]$

IJESRT is licensed under a Creative Commons Attribution 4.0 International License.

- $\frac{1}{23}[89y_{4n+5}-525x_{4n+5}+356y_{2n+3}-2100x_{2n+3}+138]$
- $\frac{1}{138}[89y_{4n+6}-6265x_{4n+5}+356y_{2n+4}-25060x_{2n+3}+828]$
- $\frac{1}{46}[353y_{4n+5}-175x_{4n+6}+1412y_{2n+3}-700x_{2n+4}+276]$
- $\frac{1}{23}[1059y_{4n+6}-6265x_{4n+6}+4236y_{2n+4}-25060x_{2n+4}+138]$
- $\frac{1}{23}[179y_{4n+5}-15y_{4n+6}+716y_{2n+3}-60y_{2n+4}+138]$

5. Each of the following expression represents a Quintic Integer:

- $\frac{1}{23}[9y_{5n+5}-35x_{5n+5}+45y_{3n+3}-175x_{3n+3}+90y_{n+1}-350x_{n+1}]$
- $\frac{1}{23}[9x_{5n+6}-89x_{5n+5}-45x_{3n+4}-445x_{3n+3}+90x_{n+2}-890x_{n+1}]$
- $\frac{1}{92}[3x_{5n+7}-353x_{5n+5}+15x_{3n+5}-1765x_{3n+3}+30x_{n+3}-3530x_{n+1}]$
- $\frac{1}{46}[3y_{5n+6}-175x_{5n+5}+15y_{3n+4}-875x_{3n+3}+30y_{n+2}-1750x_{n+1}]$
- $\frac{1}{1633}[9y_{5n+7}-6275x_{5n+5}+45y_{3n+5}-31325x_{3n+3}+90y_{n+3}-62650x_{n+1}]$
- $\frac{1}{138}[89y_{5n+5}-35x_{5n+6}+490y_{3n+3}-175x_{3n+4}+890y_{n+1}-350x_{n+2}]$
- $\frac{1}{1633}[1059y_{5n+5}-35x_{5n+7}+5295y_{3n+3}-175x_{3n+5}+10590y_{n+1}-350x_{n+3}]$
- $\frac{1}{23}[15y_{5n+5}-y_{5n+6}+75y_{3n+3}-5y_{3n+4}+150y_{n+1}-10y_{n+2}]$
- $\frac{1}{276}[179y_{5n+5}-y_{5n+7}+895y_{3n+3}-5y_{3n+5}+1790y_{n+1}-10y_{n+3}]$
- $\frac{1}{23}[89x_{5n+7}-1059x_{5n+6}+445x_{3n+5}-5295x_{3n+4}+890x_{n+3}-10590x_{n+2}]$
- $\frac{1}{23}[89y_{5n+6}-525x_{5n+6}+445y_{3n+4}-2625x_{3n+4}+890y_{n+2}-5250x_{n+2}]$
- $\frac{1}{138}[89y_{5n+7}-6265x_{5n+6}+445y_{3n+5}-31325x_{3n+4}+890y_{n+3}-62650x_{n+2}]$
- $\frac{1}{46}[353x_{5n+6}-175x_{5n+7}+1765y_{3n+4}-875x_{3n+5}+3542y_{n+2}-1750x_{n+3}]$
- $\frac{1}{23}[1059y_{5n+7}-6265x_{5n+7}+5295y_{3n+5}-31325x_{3n+5}+10590y_{n+3}-62650x_{n+3}]$
- $\frac{1}{23}[179y_{5n+6}-15y_{5n+7}+895y_{3n+4}-75y_{3n+5}+1790y_{n+2}-150y_{n+3}]$

IJESRT is licensed under a Creative Commons Attribution 4.0 International License.

3. REMARKABLE OBSERVATIONS

3.1 Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbolas which are presented in Table: 2 below:

Table: 2 Hyperbolas

S.NO	Hyperbola	(X,Y)
1	$Y^2-35X^2=2116$	$(9x_{n+1}-y_{n+1}, 9y_{n+1}-35x_{n+1})$
2	$Y^2-35X^2=2116$	$(15x_{n+1}-x_{n+2}, 9x_{n+2}-89x_{n+1})$
3	$Y^2-35X^2=304704$	$(179x_{n+1}-x_{n+3}, 9x_{n+3}-1059x_{n+1})$
4	$Y^2-35X^2=76176$	$(89x_{n+1}-y_{n+2}, 9y_{n+2}-525x_{n+1})$
5	$Y^2-35X^2=10666756$	$(1059x_{n+1}-y_{n+3}, 9y_{n+3}-6265x_{n+1})$
6	$Y^2-35X^2=76176$	$(9x_{n+2}-15y_{n+1}, 89y_{n+1}-35x_{n+2})$
7	$Y^2-35X^2=10666756$	$(9x_{n+3}-179y_{n+1}, 1059y_{n+1}-35x_{n+3})$
8	$Y^2-35X^2=2592100$	$(9y_{n+2}-89y_{n+1}, 525y_{n+1}-35y_{n+2})$
9	$Y^2-35X^2=373262400$	$(9y_{n+3}-1059y_{n+1}, 6265y_{n+1}-35y_{n+3})$
10	$Y^2-35X^2=2116$	$(179x_{n+2}-15x_{n+3}, 89x_{n+3}-1059x_{n+2})$
11	$Y^2-35X^2=2116$	$(89x_{n+2}-15y_{n+2}, 89y_{n+2}-525x_{n+2})$
12	$Y^2-35X^2=76176$	$(1059x_{n+2}-15y_{n+3}, 89y_{n+3}-6265x_{n+2})$
13	$Y^2-35X^2=76176$	$(89x_{n+3}-179y_{n+2}, 1059y_{n+2}-525x_{n+3})$
14	$Y^2-35X^2=2116$	$(1059x_{n+3}-179y_{n+3}, 1059y_{n+3}-6265x_{n+3})$
15	$Y^2-35X^2=2592100$	$(89y_{n+3}-1059y_{n+2}, 6265y_{n+2}-525y_{n+3})$

3.2 Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabolas which are presented in Table: 3 below:

Table: 3 Parabolas

S.NO	Parabola	(X,Y)
1	$23Y-35X^2=2116$	$(9x_{n+1}-y_{n+1}, 9y_{2n+2}-35x_{2n+2}+46)$
2	$23Y-35X^2=2116$	$(15x_{n+1}-x_{n+2}, 9x_{2n+3}-89x_{2n+2}+46)$
3	$276Y-35X^2=304704$	$(179x_{n+1}-x_{n+3}, 9x_{2n+4}-1059x_{2n+2}+552)$
4	$138Y-35X^2=76176$	$(89x_{n+1}-y_{n+2}, 9y_{2n+3}-525x_{2n+2}+276)$
5	$1633Y-35X^2=10666756$	$(1059x_{n+1}-y_{n+3}, 9y_{2n+4}-6265x_{2n+2}+3266)$
6	$138Y-35X^2=76176$	$(9x_{n+2}-15y_{n+1}, 89y_{2n+2}-35x_{2n+3}+276)$
7	$1633Y-35X^2=10666756$	$(9x_{n+3}-179y_{n+1}, 1059y_{2n+2}-35x_{2n+4}+3266)$

IJESRT is licensed under a Creative Commons Attribution 4.0 International License.

8	$805Y - 35X^2 = 2592100$	$(9y_{n+2} - 89y_{n+1}, 525y_{2n+2} - 35y_{2n+3} + 1610)$
9	$9660Y - 35X^2 = 373262400$	$(9y_{n+3} - 1059y_{n+1}, 6265y_{2n+2} - 35y_{2n+4} + 19320)$
10	$23Y - 35X^2 = 2116$	$(179x_{n+2} - 15x_{n+3}, 89x_{2n+4} - 1059x_{2n+3} + 46)$
11	$23Y - 35X^2 = 2116$	$(89x_{n+2} - 15y_{n+2}, 89y_{2n+3} - 525x_{2n+3} + 46)$
12	$138Y - 35X^2 = 76176$	$(1059x_{n+2} - 15y_{n+3}, 89y_{2n+4} - 6265x_{2n+3} + 276)$
13	$138Y - 35X^2 = 76176$	$(89x_{n+3} - 179y_{n+2}, 1059y_{2n+3} - 525x_{2n+4} + 276)$
14	$23Y - 35X^2 = 2116$	$(1059x_{n+3} - 179y_{n+3}, 1059y_{2n+4} - 6265x_{2n+4} + 46)$
15	$805Y - 35X^2 = 2592100$	$(89y_{n+3} - 1059y_{n+2}, 6265y_{2n+3} - 525y_{2n+4} + 1610)$

4. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the hyperbola; represented by positive Pell equation is given by $y^2 = 35x^2 + 46$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of positive Pell equations and determine their integer solutions along with suitable properties.

REFERENCES

[1] Disckson L E . "History of theory of number", Chelsea publishing co., New York, 1952, 2.
[2] Mordel L J. Diophantine Equations, Academic Press, London, 1969.
[3] S.J.Telang. "Number Theory", Tata mcgraw Hill Publishing company ltd, New Delhi, 2000.
[4] David Burton, "Elementary Number Theory", Tata Mcgraw Hill Publishing Company ltd, New Delhi, 2002.
[5] M.A.Gopalan, G.Janaki. Observations on $y^2 = 3x^2 + 1$ Acta Ciencia Indica, 34(2):
[6] 639-696:2008.
[7] M.A.Gopalan, G.Sangeetha. ARemarkable Observation On $y^2 = 10x^2 + 1$, Impact Journal Of Science and Technology, 4:103-106:2010.
[8] M.A.Gopalan, R.Palanikumar. Observation On $y^2 = 12x^2 + 1$, Antarctica J. Math , 8(2): 149-152;2011.
[9] M.A. Gopalan, G.Sumathi and S.Vidhyalakshmi, Oservations On the Hyperbola, $y^2 = 24x^2 + 1$, Bessel J math,4:21-25;2013.
[10] K.Meena, M.A.Gopalan and S.Nandhini, On the binary quadratic Diophantine equation $y^2 = 68x^2 + 13$, International Journal of Advanced Education and Research,2:59-63;2017.
[11] K. Meena, S. Vidhyalakshmi, R. Sobana Devi, On the binary quadratic equation $y^2 = 7x^2 + 32$. International Journal Of Advanced Science and Research,2:18-22.2017
[12] K.Meena, M.A.Gopalan, V.Sivaranjani. On the positive Pell Equation $y^2 = 102x^2 + 33$, International Journal Of Advanced Education and Research,2:4-8;2017
[13] T.R.Usha Rani, K.Dhivya. On the Positive Pell Equation $y^2 = 14x^2 + 18$, International Journal of Acadamic Research and Development,3:43-50;2018.
[14] D.Maheswari, R. Suganya. Observations on the Pell equations $y^2 = 11x^2 + 5$, International Journal of Emerging Technologies in Engineering Research ,6:45-50;2018.

IJESRT is licensed under a Creative Commons Attribution 4.0 International License.